Induction and Inductance

A current produces a magnetic field.

That fact came as a surprise to the scientists who discovered the effect.

Perhaps even more surprising was the discovery of the reverse effect:

A magnetic field can produce an electric field that can drive a current.

This link between a magnetic field and the electric field it produces (*induces*) is now called

Faraday's law of induction.

Two simple experiments about Faraday's law of induction.

Figure 1. Shows a conducting loop connected to a sensitive ammeter.

Because there is no battery or other source of emf included, there is no current in the circuit.

However, if we move a bar magnet toward the loop, a current suddenly appears in the circuit.

The current disappears when the magnet stops.

 If we then move the magnet away, a current again suddenly appears, but now in the opposite

direction.
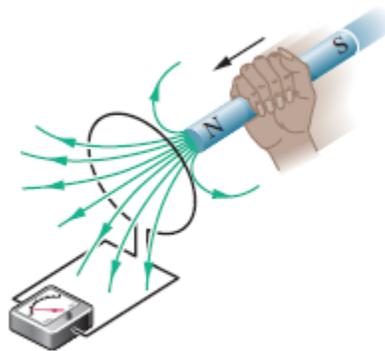
1. A current appears only if there is relative motion between the loop and the magnet; the current disappears when the relative motion between them ceases.

2. Faster motion produces a greater current.

3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current.

Moving the South Pole toward or away from the loop also causes currents, but in the reversed directions.

The current produced in the loop is called an **induced current;** the work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induced emf;** and the process of producing the current and emf is called **induction.**

Experiment 2

When the switch is open (no current), there are no field lines.

However, when we turn on the current in the right-hand loop, the increasing current builds up a magnetic field around that loop and at the left-hand loop.

While the field builds, the number of magnetic field lines through the left-hand loop increases. So, the increase in field lines through that loop apparently induces a current and an emf there.

When the current in the right-hand loop reaches a final, steady value, the number of field lines through the left-hand loop no longer changes, and the induced current and induced emf disappear.

Closing the switch
causes a current in
the left-hand loop.
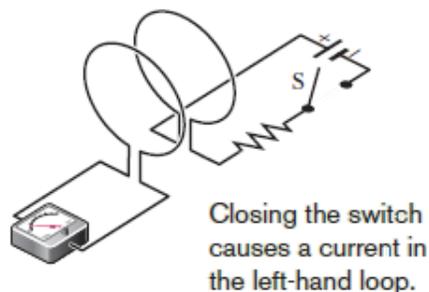
30.3   Faraday's Law of Induction

Faraday realized that an emf and a current can be induced in a loop, by changing the amount of magnetic field passing through the loop.

An emf is induced in the loop when the number of magnetic field lines that pass through the loop is changing.

The actual number of field lines passing through the loop does not matter; the values of the induced emf and induced current are determined by the rate at which that number changes.

In first experiment, as we move the north pole closer to the loop, the number of field lines passing through the loop increases.

That increase apparently causes conduction electrons in the loop to move (the induced current) and provides energy (the induced emf) for their motion.

When the magnet stops moving, the number of field lines through the loop no longer changes and the induced current and induced emf disappear.

In second experiment, when the switch is open (no current), there are no field lines.

However, when we turn on the current in the right-hand loop, the increasing current builds up a magnetic field around that loop and at the left-hand loop.

While the field builds, the number of magnetic field lines through the left-hand loop increases.

As in the first experiment, the increase in field lines through that loop apparently induces a current and an emf there.

When the current in the right-hand loop reaches a final, steady value, the number of field lines through the left-hand loop no longer changes, and the induced current and induced emf disappear.

Quantitative treatment:

To calculate the *amount of magnetic field* that passes through a loop.

Magnetic flux: Suppose a loop enclosing an area A is placed in a magnetic field .Then the **magnetic flux** through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

dA is a vector of magnitude dA that is perpendicular to a differential area dA.

Suppose that the magnetic field is perpendicular to the plane of the loop.

So the dot product in *B dA* cos 0° = *B dA*.

If the magnetic field is also uniform, then B can be brought out in front of the integral sign.

The remaining $\int A$ then gives just the area A of the loop. Thus, Eq. 30-1 reduces to

$$\Phi_B = BA \qquad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

The SI unit for magnetic flux is the tesla–square meter, which is called the weber (Wb)

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

The magnitude of the emf ( ) induced in a conducting loop is equal to the rate at which the magnetic flux ( ) through that loop changes with time.

The induced emf ( ) tends to oppose the flux change, so Faraday's law is formally written as

$$\mathscr{E} = -\frac{d\Phi_B}{dt}$$

the minus sign indicating that opposition. We often neglect the minus sign, seeking only the magnitude of the induced emf.

The total emf induced in the coil of N turns

$$\mathscr{E} = -N \frac{d\Phi_B}{dt} \quad \text{(coil of } N \text{ turns).}$$

Here are the general means by which we can change the magnetic flux through a coil:

1. Change the magnitude B of the magnetic field within the coil.

2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).

3. Change the angle between the direction of the magnetic field and the plane of the coil (for example, by rotating the coil so that field is first perpendicular to the plane of the coil and then is along that plane)

30.4 Lenz's Law

Lenz's law, named after the physicist Emil Lenz , formulated it in 1834

The direction of the current induced in a conductor by a changing magnetic field (as per Faraday's law of electromagnetic induction) is such that the magnetic field created by the induced current *opposes* the initial changing magnetic field which produced it.

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.

Figure shows Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

The magnet's motion creates a magnetic dipole that opposes the motion.
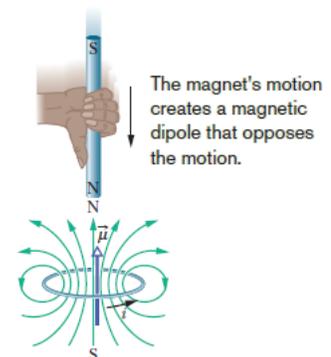
To get a feel for Lenz's law, let us apply it in two different but equivalent ways

1. *Opposition to Pole Movement.*

The approach of the magnet's north pole in Fig. 30-4 increases the magnetic flux through the loop and thereby induces a current in the loop.

We studied in ch. 29 that the loop then acts as a magnetic dipole with a south pole and a north pole, and that its magnetic dipole moment $\vec{\mu}$ is directed from south to north.

To *oppose* the magnetic flux increase being caused by the approaching magnet, the loop's north pole (and thus) must face *toward* the approaching north pole so as to repel it (Fig).

Then the curled–straight right-hand rule for $\vec{\mu}$ (in Ch. 29) tells us that the current induced in the loop must be counterclockwise as shown in Fig.

If we next pull the magnet away from the loop, a current will again be induced in the loop. Now, however, the loop will have a south pole facing the retreating north pole of the magnet, so as to oppose the retreat. Thus, the induced current will be clockwise.

2. **Opposition to Flux Change.**

In Fig. , with the magnet initially distant, no magnetic flux passes through the loop. As the north pole of the magnet then nears the loop with its magnetic field $\vec{B}$ directed downward, the flux through the loop increases.

To oppose this increase in flux, the induced current i must set up its own field $\vec{B}_{ind}$ directed upward inside the loop, as shown in Fig. 30-5a; then the upward flux of field $\vec{B}_{ind}$ opposes the increasing downward flux of field $\vec{B}$.

The curled–straight right-hand rule of Fig. 29-21 then tells us that i must be counterclockwise in Fig. 30-5a.

Note carefully that the flux of $\vec{B}_{ind}$ always opposes the change in the flux of $\vec{B}$, but that does not always mean that $\vec{B}_{ind}$ points opposite $\vec{B}$.

For example, if we next pull the magnet away from the loop in Fig. 30-4, the flux ϕ_B from the magnet is still directed downward through the loop, but it is now decreasing.

The flux of $\vec{B}_{ind}$ must now be downward inside the loop, to oppose the decrease in ϕ_B, as shown in Fig. 30-5b.Thus, $\vec{B}_{ind}$ and $\vec{B}$ are now in the same direction.

In Figs. 30-5c and d, the south pole of the magnet approaches and retreats from the loop, respectively.

Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that opposes the change.

Decreasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that opposes the change.

Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that opposes the change.

Decreasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that opposes the change.

The induced current creates this field, trying to offset the change.

The fingers are in the current's direction; the thumb is in the induced field's direction.
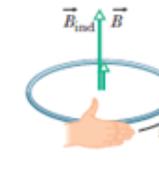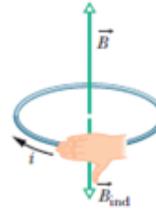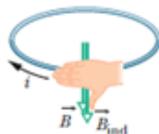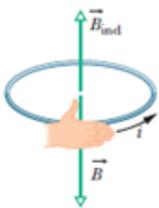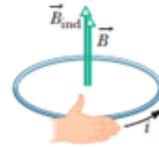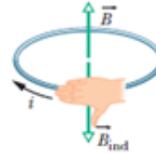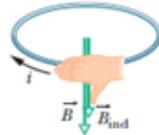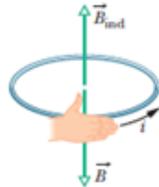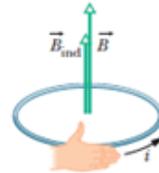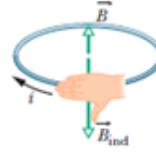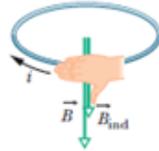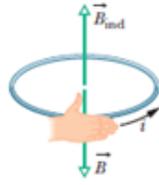
(a)   (b)   (c)   (d)

The direction of the current i induced in a loop is such that the current's magnetic field $\vec{B}_{ind}$ opposes the *change* in the magnetic field B inducing i. The field $\vec{B}_{ind}$ is always directed opposite an increasing field B (a,c) and in the same direction as a decreasing field B .The curled - straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

30.5   Induction and Energy Transfers

By Lenz's law, whether you move the magnet toward or away from the loop in Fig. 30-1, a magnetic force resists the motion, requiring your applied force to do positive work.

At the same time, thermal energy is produced in the material of the loop because of the material's electrical resistance to the current that is induced by the motion.

The energy you transfer to the closed loop + magnet system via your applied force ends up in this thermal energy. (For now, we neglect energy that is radiated away from the loop as electromagnetic waves during the induction.)

The faster you move the magnet, the more rapidly your applied force does work and the greater the rate at which your energy is transferred to thermal energy in the loop; that is, the power of the transfer is greater.

Regardless of how current is induced in a loop, energy is always transferred to thermal energy during the process because of the electrical resistance of the loop (unless the loop is superconducting).

In this fig., you pull a closed conducting loop out of a magnetic field at constant velocity. While the loop is moving, a clockwise current i is induced in the loop, and the loop segments still within the magnetic field experience forces F_1, F_2, and F_3 .

Decreasing the area decreases the flux, inducing a current.
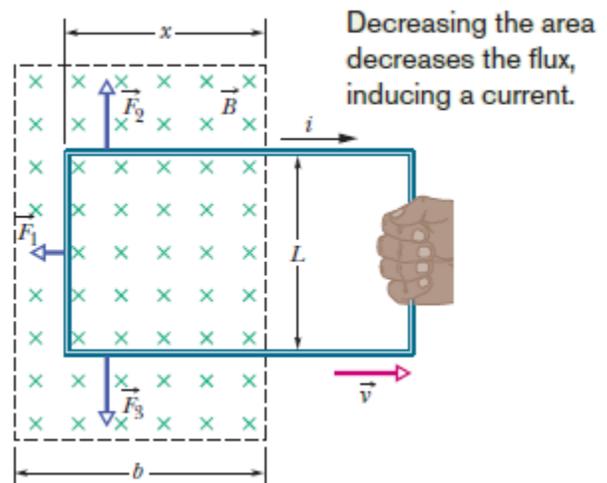
Figure shows another situation involving induced current. A rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in Fig. shows the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity.

The situation of Fig. 30-8 does not differ in any essential way from that of Fig. 30-1.

In each case a magnetic field and a conducting loop are in relative motion; in each case the flux of the field through the loop is changing with time.

It is true that in Fig. 30-1 the flux is changing because B is changing and in Fig. 30-8 the flux is changing because the area of the loop still in the magnetic field is changing, but that difference is not important.

The important difference between the two arrangements is that the arrangement of Fig. 30-8 makes calculations easier. Let us now calculate the rate at which you do mechanical work as you pull steadily on the loop in Fig. 30-8.

As you will see, to pull the loop at a constant velocity, you must apply a constant force F to the loop because a magnetic force of equal magnitude but opposite direction acts on the loop to oppose you.

$$P = FV$$

where F is the magnitude of your force. We wish to find an expression for P in terms of the magnitude B of the magnetic field and the characteristics of the loop - namely, its resistance R to current and its dimension L.

The magnitude of the flux through the loop is

$$\Phi_B = BA = BLx.$$

As x decreases, the flux decreases. Faraday's law tells us that with this flux decrease, an emf is induced in the loop. Dropping the minus sign and using above Eq., we can write the magnitude of this emf as

$$\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL\frac{dx}{dt} = BLv,$$

In which we have replaced dx/dt with v, the speed at which the loop moves

Figure 30-9 shows the loop as a circuit: induced emf ε is represented on the left, and the collective resistance R of the loop is represented on the right.
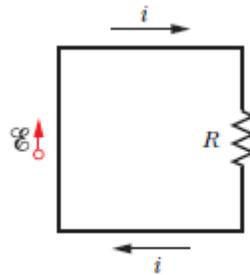
Fig. 30-9 A circuit diagram for the loop of Fig. 30-8 while the loop is moving.

The direction of the induced current i is obtained with a right-hand rule as in Fig. 30-5b for decreasing flux; applying the rule tells us that the current must be clockwise, and ε must have the same direction.

To find the magnitude of the induced current, we cannot apply the loop rule for potential differences in a circuit because, as you will see in next Section, we cannot define a potential difference for an induced emf.

However, we can apply the equation $i = \dfrac{\varepsilon}{R}$. The above eq. becomes

$$i = \frac{BLv}{R}$$

There are three segments of the loop in Fig. 30-8 carry this current through the magnetic field, sideways deflecting forces act on those segments. Deflecting force on conductor carrying current in general notation,

$$\vec{F}_d = i\vec{L} \times \vec{B}$$

In Fig. 30-8, the deflecting forces acting on the three segments of the loop are marked F_1, F_2 and F_3. Note, however, that from the symmetry, forces F_2 and F_3 are equal in magnitude and cancel. This leaves only force F_1, which is directed opposite your force F on the loop and thus is the force opposing you. So, F = - F_1

To obtain the magnitude of F_1 and noting that the angle between B and L the length vector for the left segment is 90°, we write

$$F = \mathbf{F}_1 = iLB\sin 90° = iLB$$

$$F = \frac{B^2 L^2 v}{R}$$

Because B, L, and R are constants, the speed v at which you move the loop is constant if the magnitude F of the force you apply to the loop is also constant

$$P = Fv = \frac{B^2 L^2 v^2}{R}$$

Thermal energy appears in the loop

$$P = i^2 R$$

$$P = \left(\frac{BLv}{R}\right)^2 R$$

Thus, the work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop.

Eddy Currents

Suppose we replace the conducting loop of Fig. with a solid conducting plate.

If we then move the plate out of the magnetic field as we did the loop (Fig. a), the relative motion of the field and the conductor again induces a current in the conductor.

Thus, we again encounter an opposing force and must do work because of the induced current.

With the plate, however, the conduction electrons making up the induced current do not follow one path as they do with the loop.

Instead, the electrons swirl about within the plate as if they were caught in an eddy (whirlpool) of water. Such a current is called an eddy current and can be represented, as it is in Fig. 10a, as if it followed a single path.

As with the conducting loop of rectangular coil, the current induced in the plate results in mechanical energy being dissipated as thermal energy.

The dissipation is more apparent in the arrangement of Fig. b; a conducting plate, free to rotate about a pivot, is allowed to swing down through a magnetic field like a pendulum.

Each time the plate enters and leaves the field, a portion of its mechanical energy is transferred to its thermal energy. After several swings, no mechanical energy remains and the warmed-up plate just hangs from its pivot.
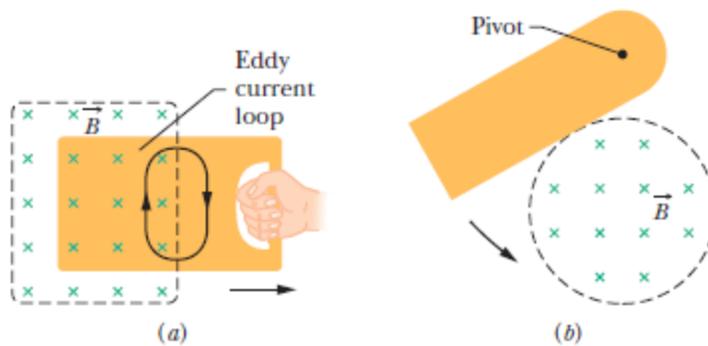
Fig. 30-10 (*a*) As you pull a solid conducting plate out of a magnetic field, *eddycurrents* are induced in the plate.A typical loop of eddy current is shown.

(*b*) A conducting plate is allowed to swing like a pendulum about a pivot and into a region of magnetic field.As it enters and leaves the field, eddy currents are induced in the plate.

30.6 Induced Electric Fields

Let us place a copper ring of radius r in a uniform external magnetic field, as in Fig. 30-11a.

The field - neglecting fringing - fills a cylindrical volume of radius R.

Suppose that we increase the strength of this field at a steady rate, perhaps by increasing - in an

appropriate way - the current in the windings of the electromagnet that produces the field.

The magnetic flux through the ring will then change at a steady rate and - by Faraday's law - an

induced emf and thus an induced current will appear in the ring.

From Lenz's law we can deduce that the direction of the induced current is counterclockwise in

Fig. 30-11a.

(*a*) If the magnetic field increases at a steady rate, a constant induced current appears, as shown, in the copper ring of radius r.
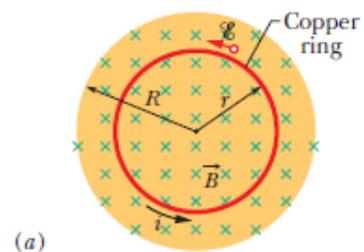
If there is a current in the copper ring, an electric field must be present along the ring because an electric field is needed to do the work of moving the conduction electrons. Moreover, the electric field must have been produced by the changing magnetic flux .This induced electric field E is just as real as an electric field produced by static charges; either field will exert a force q_0E on a particle of charge q_0.

A changing magnetic field produces an electric field.

To fix these ideas, consider Fig. b, which is just like Fig. (a) except the copper ring has been replaced by a hypothetical circular path of radius r.

We assume, as previously, that the magnetic field is increasing in magnitude at a constant rate dB/dt.

The electric field induced at various points around the circular path must - from the symmetry - be tangent to the circle, as Fig. (b) Shows.

Hence, the circular path is an electric field line. There is nothing special about the circle of radius r, so the electric field lines produced by the changing magnetic field must be a set of concentric circles, as in Fig. c.
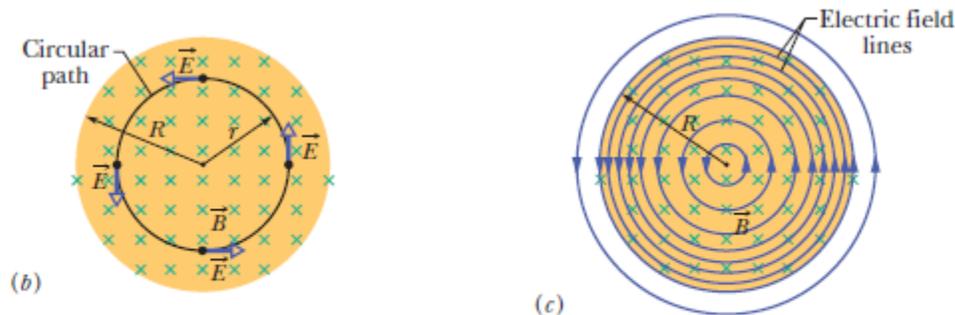
(b) An induced electric field exists even when the ring is removed; the electric field is shown at four points.
(c) The complete picture of the induced electric field, displayed as field lines.

As long as the magnetic field is increasing with time, the electric field represented by the circular field lines in Fig. 30-11c will be present. If the magnetic field remains constant with time, there will be no induced electric field and thus no electric field lines. If the magnetic field is ecreasing with time (at a constant rate), the electric field lines will still be concentric circles as in Fig. c, but they will now have the opposite direction. All this is what we have in mind when we say "A changing magnetic field produces an electric field."
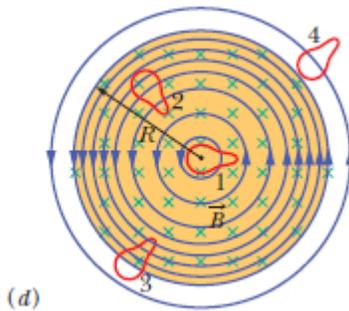
(d) Four similar closed paths that enclose identical areas. Equal emfs are induced around paths 1 and 2, which lie entirely within the region of changing magnetic field.A smaller emf is induced around path 3, which only partially lies in that region. No net emf is induced around path 4, which lies entirely outside the magnetic field.

A Reformulation of Faraday's Law

Consider a particle of charge q_0 moving around the circular path of Fig. *bas shown above*.

The work W done on it in one revolution by the induced electric field is $w = q_0 \varepsilon$ ($v = w/q_0$)

that is, the work done per unit charge in moving the test charge around the path.

From another point of view, the work is

$$W = \int \vec{F} \cdot d\vec{s} = (q_0 E)(2\pi r),$$

1

where q_0E is the magnitude of the force acting on the test charge and 2pr is the distance over which that force acts. Setting these two expressions for W equal to each other and canceling q_0, we find that

$$\mathscr{E} = 2\pi r E.$$

2

Next we rewrite Eq. 1 to give a more general expression for the work done on a particle of charge q_0 moving along any closed path:

$$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}$$

3

(The loop on each integral sign indicates that the integral is to be taken around the closed path.) Substituting q_0 for W, we find that

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}.$$

4

This integral reduces at once to Eq. 2 if we evaluate it for the special case of Fig. b.

With Eq. 4, we can expand the meaning of induced emf.

Up to this point, induced emf has meant the work per unit charge done in maintaining current due to a changing magnetic flux, or it has meant the work done per unit charge on a charged particle that moves around a closed path in a changing magnetic flux.

However, with Fig. b and Eq. 4, an induced emf can exist without the need of a current or particle: An induced emf is the sum - via integration - of quantities $\vec{E} \cdot d\vec{s}$ around a closed path, where $\vec{E}$ is the electric field induced by a changing magnetic flux and is a differential length vector along the path.

If we combine Eq. 4 with Faraday's law ($\varepsilon = - d\phi_B/dt$), we can rewrite Faraday's law as

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}.$$

5

This equation says simply that a changing magnetic field induces an electric field.
The changing magnetic field appears on the right side of this equation, the electric field on the left.

Faraday's law in the form of Eq. 5 can be applied to *any* closed path that can be drawn in a changing magnetic field. Figure d, for example, shows four such paths, all having the same shape and area but located in different positions in the changing field.

The induced emfs $\mathscr{E}\left(= \oint \vec{E} \cdot d\vec{s}\right)$ for paths 1 and 2 are equalbecause these paths lie entirely in the magnetic field and thus have the same value of $d\phi_B/dt$.

This is true even though the electric field vectors at points along these paths are different, as indicated by the patterns of electric field lines in the figure. For path 3 the induced emf is smaller because the enclosed flux ϕ_B (hence $d\phi_B/dt$) is smaller, and for path 4 the induced emf is zero even though the electric field is not zero at any point on the path.

A New Look at Electric Potential

Induced electric fields are produced not by static charges but by a changing magnetic flux. Although electric fields produced in either way exert forces on charged particles, there is an important difference between them.

The simplest evidence of this difference is that the field lines of induced electric fields form closed loops, as in Fig. *c*. Field lines produced by static charges never do so but must start on positive charges and end on negative charges.

In a more formal sense, we can state the difference between electric fields produced by induction and those produced by static charges in these words:

Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

Recall previous equation of potential difference between two points i and f in an electric field

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

If *i* and *f* in above Eq. are the same point, the path connecting them is a closed loop, V_i and V_f are identical, and above Eq. reduces to

$$\oint \vec{E} \cdot d\vec{s} = 0.$$

However, when a changing magnetic flux is present, this integral is not zero but is -dϕ_B/dt, as above Eq. asserts. Thus, assigning electric potential to an induced electric field leads us to a contradiction. We must conclude that electric potential has no meaning for electric fields associated with induction.

30.7 Inductors and Inductance

We found that a capacitor can be used to produce a desired electric field. We considered the parallel-plate arrangement as a basic type of capacitor.

Similarly, an **inductor** can be used to produce a desired magnetic field. We shall consider a long solenoid (more specifically, a short length near the middle of a long solenoid) as our basic type of inductor.

If we establish a current i in the windings (turns) of the solenoid we are taking as our inductor, the current produces a magnetic flux ϕ_B through the central region of the inductor. The inductance of the inductor is then

$$L = \frac{N\Phi_B}{i} \qquad \text{(inductance defined)}$$

N is the number of turns.

The windings of the inductor are said to be linked by the shared flux, and the product $N\phi_B$ is called the magnetic flux linkage.

The inductance L is thus a measure of the flux linkage produced by the inductor per unit of current.

Because the SI unit of magnetic flux is the tesla–square meter, the SI unit of inductance is the tesla - square meter per ampere (T-m²/A). We call this the **henry** (H), after American physicist Joseph Henry, the codiscoverer of the law of induction and a contemporary of Faraday. Thus,

$$1 \text{ henry} = 1 \text{ H} = 1 \text{ T-m}^2/\text{A}$$

Through the rest of this chapter we assume that all inductors, no matter what their geometric arrangement, have no magnetic materials such as iron in their vicinity. Such materials would distort the magnetic field of an inductor.

Inductance of a Solenoid

Consider a long solenoid of cross-sectional area A. What is the inductance per unit length near its middle?

We must calculate the flux linkage set up by a given current in the solenoid windings.

Consider a length l near the middle of this solenoid. The flux linkage there is

$$N\Phi_B = (nl)(BA)$$

$n =$ number of turns per unit length of the solenoid

$B =$ magnitude of the magnetic field within the solenoid.

The magnitude B is

$$B = \mu_0 in,$$

So

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i}$$
$$= \mu_0 n^2 lA.$$

Thus, the inductance per unit length near the center of a long solenoid is

$$\frac{L}{l} = \mu_0 n^2 A \qquad \text{(solenoid).}$$

Inductance - like capacitance - depends only on the geometry of the device.

The dependence on the square of the number of turns per unit length is to be expected.

If you, say, triple n, you not only triple the number of turns (N) but you also triple the flux ($\phi_B =$ BA $= \mu_0 i$ nA) through each turn, multiplying the flux linkage N ϕ_B and thus the inductance L by a factor of 9.

If the solenoid is very much longer than its radius, then above Eq. gives its inductance to a good approximation. This approximation neglects the spreading of the magnetic field lines near the

ends of the solenoid, just as the parallel-plate capacitor formula ($C = \varepsilon_\circ A/d$) neglects the fringing of the electric field lines near the edges of the capacitor plates.

n is a number per unit length, we can see that an inductance can be written as a product of the permeability constant $\mu_\circ$ and a quantity with the dimensions of a length. This means that $\mu_\circ$ can be expressed in the unit **henry per meter**:

$$\mu_0 = 4\pi \times 10^{-7}\,\text{T·m/A}$$
$$= 4\pi \times 10^{-7}\,\text{H/m}.$$

30.8   Self-Induction

If two coils—which we can now call inductors - are near each other, a current i in one coil produces a magnetic flux ϕ_B through the second coil. We have seen that if we change this flux by changing the current, an induced emf appears in the second coil according to Faraday's law. An induced emf appears in the first coil as well.

An induced emf ε_L appears in any coil in which the current is changing.

Fig. 30-13 If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf ε_L will appear in the coil while the current is changing.
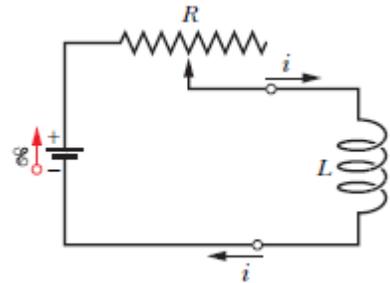
This process (see Fig. 30-13) is called self-induction, and the emf that appears is called a self-induced emf. It obeys Faraday's law of induction just as other induced emfs do.

For any inductor, we can write that

$$N\Phi_B = Li.$$

1

Faraday's law tells us that

$$\mathcal{E}_L = -\frac{d(N\Phi_B)}{dt}$$

2

By combining Eqs. 1 & 2

$$\mathcal{E}_L = -L\frac{di}{dt} \qquad \text{(self-induced emf)}.$$

3

The magnitude of the current has no influence on the magnitude of the induced emf; only the rate of change of the current counts.

You can find the direction of a self-induced emf from Lenz's law. The minus sign in Eq. 3 indicates that - as the law states - the self-induced emf $\mathcal{E}_L$ has the orientation such that it opposes the change in current i. We can drop the minus sign when we want only the magnitude of $\mathcal{E}_L$.

Suppose that, as in below Fig. a, you set up a current i in a coil and arrange to have the current increase with time at a rate di/dt. In the language of Lenz's law, this increase in the current is the "change" that the self-induction must oppose. For such opposition to occur, a self-induced emf must appear in the coil, pointing - as the figure shows - so as to oppose the increase in the current. If you cause the current to decrease with time, as in Fig. b, the self-induced emf must point in a direction that tends to oppose the decrease in the current, as the figure shows. In both cases, the emf attempts to maintain the initial condition.

The changing
current changes
the flux, which
creates an emf
that opposes
the change.
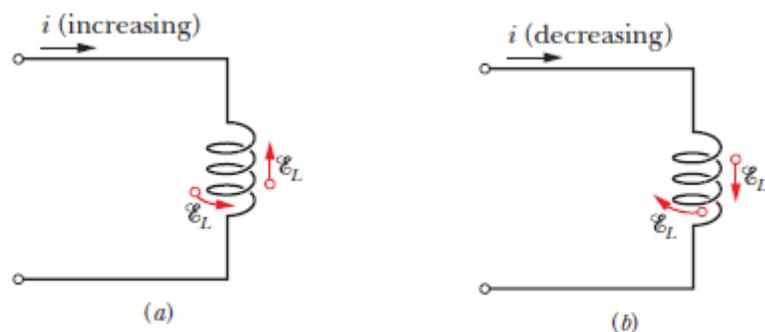
We cannot define an electric potential for an electric field (and thus for an emf) that is induced by a changing magnetic flux.

This means that when a self-induced emf is produced in the inductor of Fig. 30-13, we cannot define an electric potential within the inductor itself, where the flux is changing. However, potentials can still be defined at points of the circuit that are not within the inductor - points where the electric fields are due to charge distributions and their associated electric potentials.

Moreover, we can define a self-induced potential difference V_L across an inductor (between its terminals, which we assume to be outside the region of changing flux). For an ideal inductor (its wire has negligible resistance), the magnitude of V_L is equal to the magnitude of the self-induced emf ε_L.

If, instead, the wire in the inductor has resistance r, we mentally separate the inductor into a resistance r (which we take to be outside the region of changing flux) and an ideal inductor of self-induced emf ε_L. As with a real battery of emf ε and internal resistance r, the potential difference across the terminals of a real inductor then differs from the emf. Unless otherwise indicated, we assume here that inductors are ideal.

30-9

RL Circuits

If we suddenly introduce an emf ε into a single-loop circuit containing a resistor R and a capacitor C, the charge on the capacitor does not build up immediately to its final equilibrium value $C\varepsilon$ but approaches it in an exponential fashion

$$q = C\varepsilon(1 - e^{-t/\tau_C}).$$

The rate at which the charge builds up is determined by the capacitive time constant τ_c

$$\tau_C = RC.$$

If we suddenly remove the emf from this same circuit, the charge does not immediately fall to zero but approaches zero in an exponential fashion

$$q = q_0 e^{-t/\tau_C}.$$

The time constant τ_c describes the fall of the charge as well as its rise.

An analogous slowing of the rise (or fall) of the current occurs if we introduce an emf ε into (or remove it from) a single-loop circuit containing a resistor R and an inductor L.

When the switch S in Fig. 30-15 is closed on a, for example, the current in the resistor starts to rise. If the inductor were not present, the current would rise rapidly to a steady value ε/R. Because of the inductor, however, a selfinduced emf ε_L appears in the circuit; from Lenz's law, this emf opposes the rise of the current, which means that it opposes the battery emf ε in polarity. Thus, the current in the resistor responds to the difference between two emfs, a constant ε due to the battery and a variable ε_L (= -L di/dt) due to self-induction. As long as ε_L is present, the current will be less than ε/R.
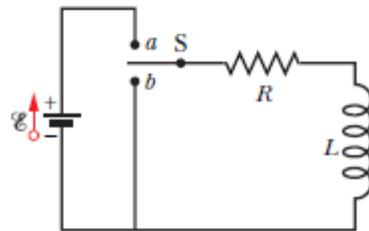
Fig. 30-15 An RL circuit. When switch S is closed on a, the current rises and approaches a limiting value ε/R.

As time goes on, the rate at which the current increases becomes less rapid and the magnitude of the self-induced emf, which is proportional to di/dt, becomes smaller. Thus, the current in the circuit approaches ε/R asymptotically.

Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.
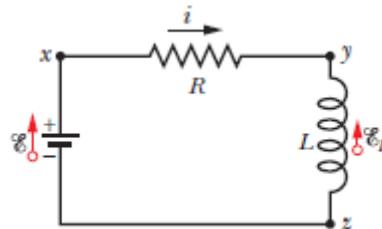
Fig. 30-16   The circuit of Fig. 30-15 with the switch closed on *a*. We apply the loop rule for the circuit clockwise, starting at *x*.

With the switch S in Fig. 30-15 thrown to *a*, the circuit is equivalent to that of Fig. 30-16. Let us apply the loop rule, starting at point *x* in this figure and moving clockwise around the loop along with current *i*.

1. Resistor. Because we move through the resistor in the direction of current i, the electric potential decreases by iR. Thus, as we move from point x to point y, we encounter a potential change of $-iR$.

2. Inductor. Because current i is changing, there is a self-induced emf ε_L in the inductor. The magnitude of ε_L is given as L di/dt. The direction of ε_L is upward in Fig. 30-16 because current i is downward through the inductor and increasing. Thus, as we move from point y to point z, opposite the direction of ε_L, we encounter a potential change of -L di/dt.

3. Battery. As we move from point z back to starting point x, we encounter a potential change of $+\varepsilon$ due to the battery's emf.

Thus, the loop rule gives us

$$-iR - L\frac{di}{dt} + \varepsilon = 0$$

$$L\frac{di}{dt} + Ri = \varepsilon \qquad (RL\ circuit). \qquad\qquad 1$$

Equation 1 is a differential equation involving the variable i and its first derivative di/dt.

To solve it, we seek the function $i(t)$ such that when $i(t)$ and its first derivative are substituted in

Eq. 1, the equation is satisfied and the initial condition i(0) " 0 is satisfied.

Equation 1 and its initial condition are of exactly the form of Eq. 27-32 for an RC circuit, with i

replacing q, L replacing R, and R replacing $1/C$. The solution of Eq. 1 must then be of exactly the

form of Eq. 27-33 with the same replacements.

That solution is

$$i = \frac{\varepsilon}{R}(1 - e^{-Rt/L}), \qquad\qquad 2$$

$$i = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L}) \qquad (\text{rise of current}). \qquad\qquad 3$$

We can write

The **inductive time constant,** is given by

$$\tau_L = \frac{L}{R} \quad \text{(time constant)}. \qquad\qquad 4$$

Let's examine Eq. 3 for just after the switch is closed (at time t = 0) and for a time long after the switch is closed (t→∞). If we substitute t = 0 into Eq.3, the exponential becomes e^{-0} = 1.Thus, Eq. 3 tells us that the current is initially i = 0, as we expected. Next, if we let t go to ∞, then the exponential goes to $e^{-\infty}$ = 0. Thus, Eq. 3 tells us that the current goes to its equilibrium value of ε/R.

We can also examine the potential differences in the circuit. For example, Fig. 30-17 shows how the potential differences V_R (= iR) across the resistor and V_L (= L di/dt) across the inductor vary with time for particular values of ε, L, and R. Compare this figure carefully with the corresponding figure for an RC circuit (Fig. 27-16).

The resistor's potential difference turns on.
The inductor's potential difference turns off.

(a)

(b)
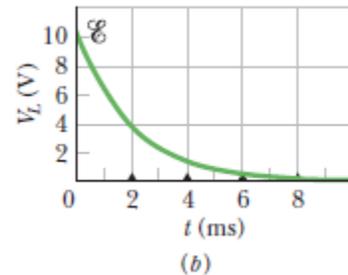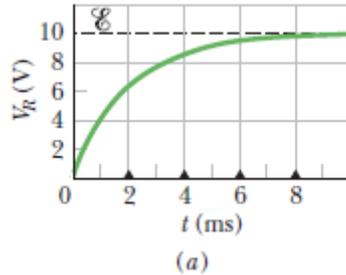
To show that the quantity τ_L(= L/R) has the dimension of time, we convert from henries per ohm as follows:

$$1\frac{H}{\Omega} = 1\frac{H}{\Omega}\left(\frac{1\,V\cdot s}{1\,H\cdot A}\right)\left(\frac{1\,\Omega\cdot A}{1\,V}\right) = 1\,s.$$

The first quantity in parentheses is a conversion factor based on Eq. 30-35, and the second one is a conversion factor based on the relation $V = iR$.

The physical significance of the time constant follows from Eq. 30-41. If we put $t = \tau_L = L/R$ in this equation, it reduces to

$$i = \frac{\mathscr{E}}{R}(1 - e^{-1}) = 0.63\frac{\mathscr{E}}{R}. \qquad (30\text{-}43)$$

Thus, the time constant τ_L is the time it takes the current in the circuit to reach about 63% of its final equilibrium value $\mathscr{E}/R$. Since the potential difference V_R across the resistor is proportional to the current i, a graph of the increasing current versus time has the same shape as that of V_R in Fig. 30-17a.

If the switch S in Fig. 30-15 is closed on a long enough for the equilibrium current $\mathscr{E}/R$ to be established and then is thrown to b, the effect will be to remove the battery from the circuit. (The connection to b must actually be made an instant before the connection to a is broken. A switch that does this is called a *make-before-break* switch.) With the battery gone, the current through the resistor will decrease. However, it cannot drop immediately to zero but must decay to zero over time. The differential equation that governs the decay can be found by putting $\mathscr{E} = 0$ in Eq. 30-39:

$$L\frac{di}{dt} + iR = 0. \qquad (30\text{-}44)$$

By analogy with Eqs. 27-38 and 27-39, the solution of this differential equation that satisfies the initial condition $i(0) = i_0 = \mathscr{E}/R$ is

$$i = \frac{\mathscr{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L} \qquad \text{(decay of current).} \qquad (30\text{-}45)$$

30-10 **Energy Stored in a Magnetic Field**

When we pull two charged particles of opposite signs away from each other, we say that the resulting electric potential energy is stored in the electric field of the particles.

We get it back from the field by letting the particles move closer together again. In the same way we say energy is stored in a magnetic field, but now we deal with current instead of electric charges.

To derive a quantitative expression for that stored energy, consider again Fig. 30-16, which shows a source of emf ε connected to a resistor R and an inductor L. Equation 30-39, restated here for convenience,
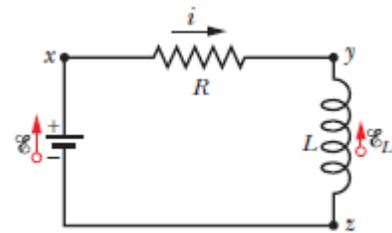
$$L\frac{di}{dt} + Ri = \mathcal{E} \qquad (RL\text{ circuit}).$$ 1

Multiply each side by i, we obtain

$$\mathcal{E}i = Li\frac{di}{dt} + i^2R.$$ 2

Which has the following physical interpretation in terms of the work done by the battery and the resulting energy transfers:

1. If a differential amount of charge dq passes through the battery of emf ε in Fig. above in time dt, the battery does work on it in the amount εdq. The rate at which the battery does work is (ε dq)/dt, or ε i. Thus, the left side of Eq. 2 represents the rate at which the emf device delivers energy to the rest of the circuit.

2. The rightmost term in Eq. 2 represents the rate at which energy appears as thermal energy in the resistor.

3. Energy that is delivered to the circuit but does not appear as thermal energy must, by the conservation-of-energy hypothesis, be stored in the magnetic field of the inductor. Because Eq. 2 represents the principle of conservation of energy for RL circuits, the middle term must represent the rate dU_B/dt at which magnetic potential energy U_B is stored in the magnetic field.

Thus

$$\frac{dU_B}{dt} = Li\frac{di}{dt}. \qquad\qquad 3$$

We can write this as

$$dU_B = Li\, di.$$

Integrating yields

$$\int_0^{U_B} dU_B = \int_0^i Li\, di$$

$$U_B = \tfrac{1}{2}Li^2 \qquad \text{(magnetic energy)}, \qquad\qquad 4$$

which represents the total energy stored by an inductor L carrying a current i. Note the similarity in form between this expression and the expression for the energy stored by a capacitor with capacitance C and charge q; namely,

$$U_E = \frac{q^2}{2C}. \qquad\qquad 5$$

(The variable i^2 corresponds to $q2$, and the constant L corresponds to $1/C$)

30-11 Energy Density of a Magnetic Field

Consider a length l near the middle of a long solenoid of cross-sectional area A carrying current i; the volume associated with this length is Al. The energy U_B stored by the length l of the solenoid must lie entirely within this volume because the magnetic field outside such a solenoid is approximately zero.

Moreover, the stored energy must be uniformly distributed within the solenoid because the magnetic field is (approximately) uniform everywhere inside.

Thus, the energy stored per unit volume of the field is

$$u_B = \frac{U_B}{Al}$$

or, since

$$U_B = \tfrac{1}{2}Li^2,$$

we have

$$u_B = \frac{Li^2}{2Al} = \frac{L}{l}\frac{i^2}{2A}. \qquad\qquad 1$$

Here L is the inductance of length l of the solenoid.

Substituting for $L/l = \quad \frac{L}{l} = \mu_0 n^2 A$

$$u_B = \tfrac{1}{2}\mu_0 n^2 i^2, \qquad\qquad 2$$

where n is the number of turns per unit length. From Eq. ($B = \mu_0 in$) we can write this energy density as

$$u_B = \frac{B^2}{2\mu_0} \qquad \text{(magnetic energy density)} \qquad\qquad 3$$

This equation gives the density of stored energy at any point where the magnitude of the magnetic field is B. Even though we derived it by considering the special case of a solenoid, Eq. 3 holds for all magnetic fields, no matter how they are generated. The equation is comparable to Eq. 25-25,

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2$$

which gives the energy density (in a vacuum) at any point in an electric field. Note that both u_B and u_E are proportional to the square of the appropriate field magnitude, B or E.

30-12   Mutual Induction

We saw earlier that if two coils are close together, a steady current i in one coil will set up a magnetic flux ϕ through the other coil (*linking* the other coil). If we change i with time, an emf ε given by Faraday's law appears in the second coil; we called this process *induction.*

 We could better have called it **mutual induction,** to suggest the mutual interaction of the two coils and to distinguish it from *self-induction,* in which only one coil is involved.

Let us look a little more quantitatively at mutual induction. Figure 30-19a shows two circular close-packed coils near each other and sharing a common central axis. With the variable resistor set at a particular resistance R, the battery produces a steady current i_1 in coil 1.This current creates a magnetic field represented by the lines B_1 of in the figure. Coil 2 is connected to a sensitive meter but contains no battery; a magnetic flux ϕ_{21} (the flux through coil 2 associated with the current in coil 1) links the N_2 turns of coil 2.
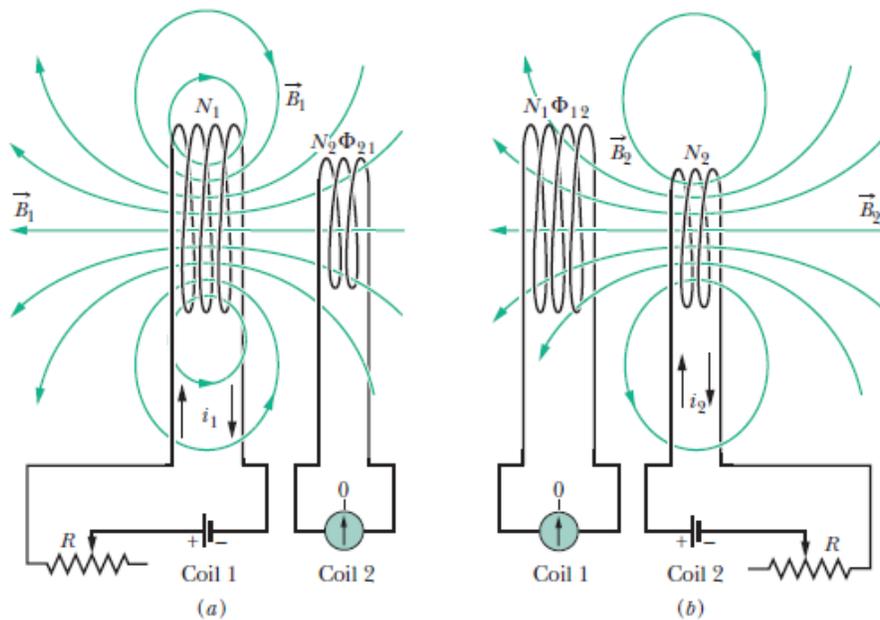
Fig. 30-19   Mutual induction. (a) The magnetic field $\vec{B}_1$ produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an emf is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.

We define the mutual inductance M_{21} of coil 2 with respect to coil 1 as

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1},$$ 1

which has the same form as

$$L = N\Phi/i,$$ 2

Eq. 1 can be written as

$$M_{21}i_1 = N_2 \Phi_{21}.$$ 3

If we cause i_1 to vary with time by varying R, we have

$$M_{21}\frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt}.$$ 4

The right side of this equation is, according to Faraday's law, just the magnitude of the emf ε_2 appearing in coil 2 due to the changing current in coil 1.Thus, with a minus sign to indicate direction,

$$\mathscr{E}_2 = -M_{21}\frac{di_1}{dt},$$ 5

which you should compare with Eq. 30-35 for self-induction ($\varepsilon = -L\, di/dt$).

Let us now interchange the roles of coils 1 and 2, as in Fig. 30-19b; that is, we set up a current i_2 in coil 2 by means of a battery, and this produces a magnetic flux ϕ_{12} that links coil 1.If we change i_2 with time by varying R, we then have, by the argument given above,

$$\mathscr{E}_1 = -M_{12}\frac{di_2}{dt}.$$ 6

Thus, we see that the emf induced in either coil is proportional to the rate of change of current in the other coil. The proportionality constants M_{21} and M_{12} seem to be different. We assert,

without proof, that they are in fact the same so that no subscripts are needed. (This conclusion is true but is in no way obvious.) Thus, we have

$$M_{21} = M_{12} = M, \qquad\qquad 7$$

so eq. 5 and 6 can be wriiten as

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \qquad\qquad 8$$

$$\mathcal{E}_1 = -M\frac{di_2}{dt}. \qquad\qquad 9$$